THE POSSIBILITY OF APPROXIMATING THE BOUNDARY OF A NONEXPANDED AXISYMMETRICAL JET TO THE ARC OF AN ELLIPSE

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An approximation of the boundary of the jet within the limits of the first "barrel" of the arc of an ellipse is proposed (on the basis of analysis of schlieren-photographs of the stream in the range M_a = 1 to 4.85, n = 1 to 10^3, $\varkappa$ = 1.3 to 1.67; β = 0 to 21°). The results of calculations of the boundary according to the proposed method are compared with results of calculation by a method of characteristics, and with experimental data.

Engineering practice often encounters problems in which it is necessary to determine the position of the boundary of a nonexpanded jet rapidly with sufficient accuracy. Simple algebraic relations which are suitable for use in a wide range of variation of the parameters M_a, n, $\varkappa$, and β are necessary for this purpose.

The use of the method of characteristics [1, 2] is limited owing to its laboriousness, and the existing approximate methods of calculating the boundary [3-5] are either accurate within a narrow range of variation of the parameters, or require cumbersome calculations.

An attempt was made by a more simple method to obtain a curve which approximately represented the boundary of the jet, in [4]. Here it is assumed that on the initial part its boundary can be represented by the arc of a circle which passes through the exit section of the nozzle and the point of the maximum diameter of the boundary. The center of this arc is situated on a normal to the boundary of the jet at the exit section of the nozzle. However, as the experiment shows, the curvature of the boundary of the jet varies along its length and therefore approximation of the boundary of the jet by the circular arc, as proposed in [4], and also in [3] can lead to considerable errors, especially in the case of high values n > 20. Analysis of schlieren-photographs obtained on an experimental test rig, which is described in [6] showed that in the range M_a = 1 to 4.85, n = 1 to 10^3, $\varkappa$ = 1.3-1.67, and β = 0 to 21° the boundary of the jet within the limits of the first "barrel" can be approximated by the arc of the ellipse; its axes, and also the coordinates of the center are represented in the form of simple relationships by M_a, n, $\varkappa$, and β if the experimental data for the position and magnitude of the maximum diameter of the first "barrel" obtained by the authors are used.

As the experiment has shown in the case of $\varkappa$ = 1.3 to 1.67, and n = 1 to 10^3, the maximum diameter of the jet decreases with an increase in the temperature T_0 [6], but, starting with T_0 = 600°K, this decrease slows down, and a further increase of T_0 up to 700 to 750°K does not lead to a noticeable variation of the maximum diameter. This evidently associated with a decrease in the influence of condensation in the jet, and at low values of n it disappears completely. The experiment did not show a noticeable influence of M_a on the maximum diameter; the main factors which determine its magnitude are $\varkappa$, n, and β. For heated jets (T_0 = 700-750°K) the experimental data for the relative maximum diameter are expressed by two relationships:

for n ≤ 20

$$\frac{D_{\max}}{d_a} = \frac{1.3}{\varkappa(\cos\beta)^2} n^{0.45}; \qquad (1)$$

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for n > 20

$$\frac{D_{max}}{d_a} = \frac{2}{\varkappa (\cos \beta)^2} n^{0.43}. \tag{2}$$

The distance from the exit section of the nozzle to the maximum of the first "barrel" can be determined from the empirical ratio

$$L_{max} = (0.75 - 0.85)\, x_0 \approx 0.8 x_0 \tag{3}$$

or in dimensionless magnitudes, using the relationship which characterizes the position of the central shock [6]:

$$\frac{L_{max}}{d_a} = 2.6 \frac{M_a^2}{M_a^2 + 1} n^{0.39}. \tag{4}$$

Now the problem of constructing the boundary of the jet is reduced to the construction of the arc of the ellipse which passes through 2 points: 1) $r = r_a$, $x = 0$, 2) $r = r_{max}$, $x = L_{max}$, in which at the first point the arc is in contact with a line which determines the exit angle of the jet on the exit section of the nozzle, and at the second point it is in contact with a line parallel to the axis of symmetry of the jet.

If three-dimensional effects are neglected, the angle of inclination of the flow on the exit section of the nozzle from the direction of the center line of the jet can be determined by the known relationships derived for a plane flow (see, for example, [7]), in which the calculations agree well with the experiment for axisymmetrical jets.

In the converted form the formula for determining δ is written

$$\delta = \sqrt{\frac{\varkappa+1}{\varkappa-1}} \left(\arcsin \sqrt{1 - \frac{\varkappa+1}{2} \frac{1}{n^{\frac{\varkappa-1}{\varkappa}} \left(1 + \frac{\varkappa-1}{2} M_a^2\right)}} - \arcsin \sqrt{\frac{\varkappa-1}{2} \frac{M_a^2 - 1}{1 + \frac{\varkappa-1}{2} M_a^2}} \right) + \arcsin \sqrt{\frac{\varkappa-1}{2} \frac{1}{n^{\frac{\varkappa-1}{\varkappa}} \left(1 + \frac{\varkappa-1}{2} M_a^2\right) - 1}} - \arcsin \frac{1}{M_a} + \beta, \tag{5}$$

where β is the half angle of the nozzle cone.

The equation of the boundary of the jet must be looked for in the form

$$\frac{(\bar{x} - m)^2}{a^2} + \frac{(\bar{r} - l)^2}{b^2} = 1$$

or

$$\bar{r} = \sqrt{b^2 - \frac{b^2}{a^2} (\bar{x} - m)^2} + l, \tag{6}$$

where

$$\bar{x} = \frac{x}{r_a}; \quad \bar{r} = \frac{r}{r_a}.$$

Hence only the positive values of the root in equation (6) are taken into consideration. The origin on the coordinates is located on the axis of the nozzle in the plane of its exit section; the axis x corresponds with the axis of symmetry of the jet.

In order to calculate the parameters a, b, m, l, four conditions are used

$$\left.\begin{array}{l} \text{I) } \bar{x} = 0; \ \frac{d\bar{r}}{d\bar{x}} = \operatorname{tg} \delta \\ \text{II) } \bar{x} = \bar{x}_{max}; \ \bar{r} = \bar{r}_{max} \\ \text{III) } \bar{x} = \bar{x}_{max}; \ \frac{d\bar{r}}{d\bar{x}} = 0 \\ \text{IV) } \bar{x} = 0; \ \bar{r} = 1 \end{array}\right\}. \tag{7}$$

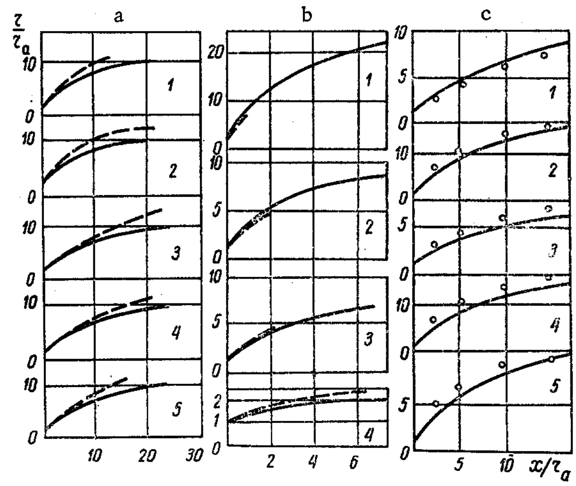

Fig. 1. Comparison of calculations of the boundary of the jet: a) according to the proposed method with calculations by a method of characteristics [1]; b) by a method of characteristics [2] for $M_a = 2$, $\varkappa = 1.25$, $\beta = 0$; and c) with the experimental data of the authors (the solid curves show the proposed method, the broken lines show the method of characteristics, and the dots show the experimental data): a) 1) $M_a = 2$; $\varkappa = 1.15$; n = 56.5; $\beta = 0$; 2) 2; 1.15; 56.5; 15°; 3) 5; 1.15; 41.6; 15°; 4) 3.1; 1.25; 62.9; $\beta = 0$; 5) 3.0; 1.15; 62.9; $\beta = 0$; b) 1) n = 500; 2) 50; 3) 30; 4) 5; c) 1) $M_a = 3.35$; $\varkappa = 1.4$; n = 70; $\beta = 8°$; 2) 2.175; 1.67; 367; 8°; 3) 4.85; 1.67; 43; 8°; 4) 2.62; 1.67; 246; 21°; 5) 1.97; 1.3; 68; 8°.

As a result of simple calculations it is possible to obtain:

$$m = \bar{x}_{\max}, \tag{8}$$

$$\frac{b^2}{a^2} = \frac{\operatorname{tg}\delta}{\bar{x}_{\max}} A, \tag{9}$$

$$l = 1 - A, \tag{10}$$

$$b = \bar{r}_{\max} - 1 + A, \tag{11}$$

where

$$A = \frac{(\bar{r}_{\max} - 1)^2}{\bar{x}_{\max} \operatorname{tg}\delta - 2(\bar{r}_{\max} - 1)}.$$

The magnitudes $\bar{r}_{max} = r_{max}/r_a$ and $\bar{x}_{max} = L_{max}/r_a$ are easily determined from the ratios (1), (2), and (4).

The relations (9)-(11) have meaning when A > 0 or $\bar{x}_{max} \tan\delta > 2(\bar{r}_{max} - 1)$, which is fulfilled for the overwhelming majority of cases.

Using the values a, b, l, m, calculated according to the relationships (8)-(11) the boundary of the jet can be built up graphically in the form of the arc of an ellipse. In Fig. 1a and b the results of calculations based on the proposed method are compared with calculations based on a method of characteristics [1], and [2], and in Fig. 1c they are compared with experimental data. Somewhat higher values of the radius of the jet in [1] are evidently explained by the fact that here the calculation was carried out without taking into account the shock wave (the "trailing" shock), which arises in the jet.

Comparison of calculations of the boundary according to the proposed method with the results of the experiment in the case of $\varkappa = 1.3$ to 1.67 showed that the arc of the ellipse satisfactorily describes the boundary of the jet on the extent of the whole length of the first "barrel" L, paying attention to the experimental relationship obtained

$$L = 1.25x_0$$

or taking [6] into account

$$\frac{L}{d_a} = 4\frac{M_a^2}{M_a^2 + 1} n^{0.39}.$$

NOTATION

M	is the machine number;
$n = P/P_\infty$	is the degree of under-expansion
$\varkappa = c_p/c_v$	is the ratio of specific heats;
β	is the half angle of the nozzle cone at exit section;
T	is the absolute temperature of the gas;
d_a, r_a	is the diameter and radius of the nozzle on the exit section;
x	is the distance from the nozzle exit plane;
r	is the radius of the jet;
D_{max}, r_{max}	are the maximum diameter and radius of the first "barrel";
L_{max}	is the distance from the nozzle exit plane to the maximum of the first "barrel" angle of inclination of the flow on the exit section of the nozzle relative to the direction of the center line of the jet;
A	is the parameter;
L	is the length of the first "barrel" of the jet;
P	is the pressure.

Subscripts

a denotes the parameters of the gas in the exit section of the nozzle;
∞ denotes the surrounding medium.

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